The Conditional CAPM and the MDAX

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1. Introduction

Several studies have tested the Sharpe-Lintner-Black Capital Asset Pricing Model (CAPM) on the American stock market (Chen, Roll, & Stephen, 1986), (Fama & French, 1992), (Jagannathan & Wang, 1996). The study by Jagannathan & Wang extended the CAPM model to divide the beta assets into conditional and unconditional returns, and proposes a Premium Labor Model (PLM) that aims to capture the conditional returns. This term paper aims to apply the PLM to the stocks listed in the German MDAX for the years 2005 to 2015, and compare the results with those from the studies above.

The MDAX

The German MDAX is a value-weighted stock index that lists the 50 companies ranked below those 30 included in the DAX. The ranking is determined by order book volume and market capitalization. Since including all German stocks would exceed the scope of this thesis, the MDAX was chosen in order to gain insight into a cross-section of the German stock market as a whole.

Capital Asset Pricing Model for the Expected Stock returns

First, we perform a test of the pure CAPM, according to the formula

$$\mathrm{E}[\mathrm{R_i}] = \gamma_0 + \gamma_{\mathrm{vw}}\beta_i^{\mathrm{vw}} \quad .$$

With expected returns $\mathrm{E}[\mathrm{R_i}]$ of the stock $\mathrm{R_i}$, individual market beta β_i^{vw}, and market premium γ_{vw}. The market beta β_i^{vw} is the dependence of the stock on the market index, expressed via

$$\beta_i^{vw} = \mathrm{Cov}(R_i, R_{\mathrm{vw}})/\mathrm{Var}[R_{\mathrm{vw}}] \quad .$$

Premium Labor Model for the Expected Stock Returns

The PLM extends the CAPM by two further economic indicators: the labor income and the market risk premium. The labor income is included in order to closer approximate the true market portfolio by including human capital; market risk premium is included to account for the conditional changes in beta β as well as returns γ. Further elaboration may be found in (Jagannathan & Wang, 1996).

The model is then expressed by the equation

$$\mathrm{E}[\mathrm{R}_{\mathrm{it}}] = c_0 + c_{vw}\beta_i^{vw} + \mathrm{c}_{\mathrm{prem}}\beta_i^{prem} + c_{labor}\beta_i^{labor} \quad .$$

Jagannathan & Wang further place emphasis on the idea that the β_i^{vw} now only expresses the unconditional stock returns, with the conditional components now extracted by β_i^{prem}.

2. Methodology

To test the two models above, we apply the same testing methodology as proposed in (Fama & French, 1992):
For each year, we firstly calculate the pre-beta of every stocks according to

$$\mathrm{R}_{\mathrm{i}} = \mathrm{c}_0 + \beta_i^{vw}\mathrm{VW} \quad ,$$

where VW is the index performance, for the previous 24 to 60 months. Stocks with less preceding data available are left out until calculations for later years.
The stocks are then sorted into portfolios, grouped by beta. This helps minimizing statistical fluctuations as well as error-in-variable problems. However, since there are only 50 stocks total in the index, a small portfolio size of five stocks each has been chosen in order to form enough portfolios.
For each of the following 12 months, we perform a cross-sectional regression using the pre-beta of each portfolio, to calculate the associated factor risk premium γ_{vw} for each individual month:

$$\mathrm{R}_{\mathrm{i}} = c_0 + \gamma_{vw}\beta_i^{vw} \quad .$$

With all variables determined, we can then predict and compare the portfolio returns according to the indicators and the CAPM.
The procedure for the PLM follows exactly the same pattern as for the CAPM model, just with the two additional indicators included in the regressions. The Portfolios are still assigned via the market betas β_i^{vw}. For further reading on the methodology, refer to (Fama & French, 1992), (Jagannathan & Wang, 1996). For the regressions, Microsoft Excel (2013) is used. Notably, in contrast to the methods recommended in (Chen Y.-S. , 2015), we use the 'LINEST' function for all regressions, which provides a more scalable method than the analysis plug-in, while yielding the same results.

3. Description of the Data

MDAX

Due to the nature of its construction, the companies in the MDAX lie closely together in market capitalization, which should lessen the need to account for size effects in the models. The largest company in the MDAX has a free float market capitalization 24.2 times higher than the smallest one. Taking the logarithm of the market capitalization, as typically done to account for size effects (Jagannathan & Wang, 1996), we yield a difference in size factors of only 1.38 – thus, we are able to handily neglect size effects as a whole.

Ideally, an index of all German stocks should be used as the market portfolio, however since no data for such an index was available, the performance of the MDAX itself will be used as a proxy for the market portfolio.

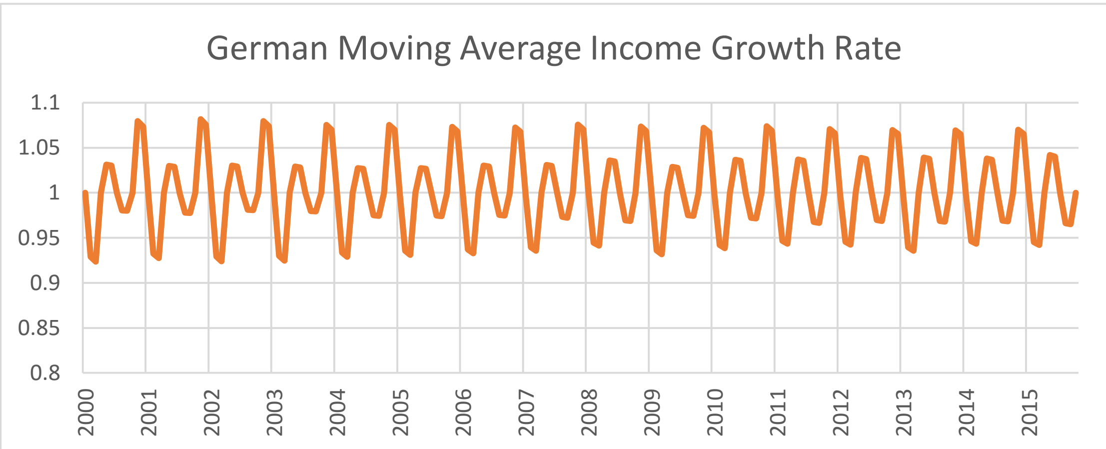
The monthly data of the stocks that are currently listed in the MDAX, as well as the index itself has been acquired for the years 2000 until 2015 from Bloomberg.

Labor Income

The quarterly nominal labor income index (Nettolohnindex) published by the Federal Statistical Office of Germany is taken as a proxy for the national labor income. Furthermore, a moving average growth rate delayed by one month is used, as proposed in (Jagannathan & Wang, 1996):

$$R_t^{\text{labor}} = [\text{L}_{\text{t}-1} + L_{t-2}]/[L_{t-2} + L_{t-3}]$$

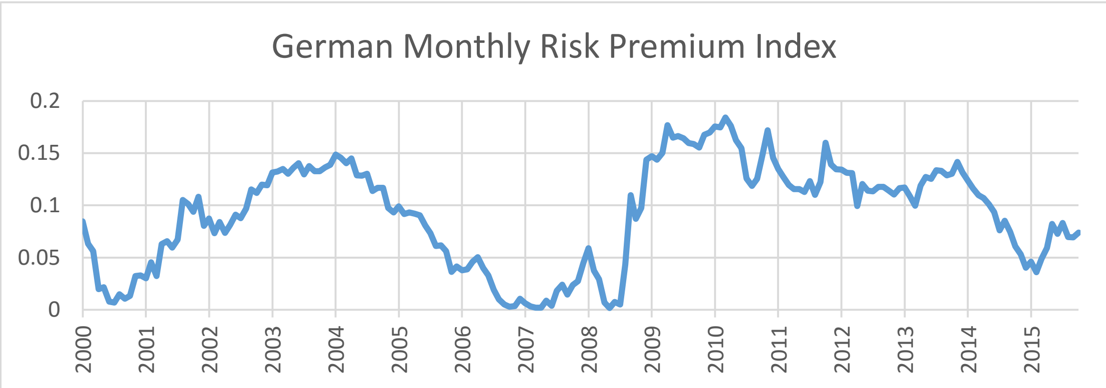
This helps in smoothing out the quarterly data over the whole year, and accounts for the fact that the labor income may only have a delayed effect on the market portfolio. The resulting index is displayed in the chart below.



Clearly visible is the strong seasonal component of the labor income, with a minimum in the first quarter and a maximum in the fourth quarter of each year. One notable contributing factor is the Christmas bonus typically paid out as a 13th monthly salary in December.

4. Risk Premium

As proposed by (Stock & Watson, 1989), we use the difference between returns of 10 year German government bonds and 2 year German government bonds as an index for the risk premium and forecaster of the business cycle. The data is downloaded from Bloomberg. The data for the 2 year government bond seemed to lag behind the one for the 10 year bond, which was adjusted for in the calculation. The resulting index is displayed below.



On first look, the index provides a very good measure of the risk premium, with the risk premium statistic sharply rising during the financial instabilities in the years 2000/2001 as well as 2008, as one would expect.

Note that in contrast to the other indicators used for the regression, we use the absolute value of the risk premium for calculations, not the growth rate. This follows the procedure presented in (Jagannathan & Wang, 1996).

5. Empirical Results

The betas of the portfolios as calculated via the CAPM, averaged over the years 2005 to 2015 are shown in the table below.

Time-Averaged Betas of Portfolios CAPM								
Portfolio	a	b	c	d	e	f	g	h
β_{vw}	1.68	1.33	1.15	1.06	0.92	0.80	0.63	0.40

Because not all firms currently in the MDAX had tradable stocks before 2005, only 6 portfolios are created for 2005, which grows to 8 until 2015. The new portfolios are added for intermediate betas as portfolio 'd' and 'e'. The averaged beta of portfolio 'a' is larger by 1.28 than the one of the lowest-beta portfolio 'h'.

By applying the extensions of the PLM, we achieve the time-averaged betas presented below:

Time-Averaged Betas of Portfolios PLM								
Portfolio	a	b	c	d	e	f	g	h
β_{vw}	1.66	1.32	1.14	1.03	0.87	0.78	0.63	0.38
β_{labor}	-0.05	-0.17	-0.09	-0.05	0.06	-0.01	0.02	0.07
β_{prem}	0.13	0.09	0.05	0.15	0.16	0.03	0.00	0.08

The averaged β_{vw} of the portfolios remain almost the same even while extracting the components β_{labor} and β_{prem}. The latter two interestingly do not stay consistent in sign, some being positive while others are negative.

The evaluation data of the two models is given in the following tables:

Static CAPM					
Coefficient	c_0	c_{vw}	c_{prem}	c_{labor}	R^2
Estimate	1.01	0.0044			23.29
t-value	30.57	0.14			
t-value via VAR	2.22	0.011			

PLM					
Coefficient	c_0	c_{vw}	c_{prem}	c_{labor}	R^2
Estimate	1.01	0.0046	-0.037	0.0044	55.49
t-value	20.56	0.11	-0.14	0.025	
t-value via VAR	0.39	0.0079	-0.0077	0.0017	

The intercept c_0 is close to 1, predicting zero excess returns when taking no beta risk, just as the two theories predict. This is a much different result from Jagannathan & Wang, where the c_0 varied much more. Consequently, the t-value reaches significant values of over 30 and over 20 respectively in our calculations.

Note that this t-value is simply calculated by averaging the t-value of each month, which is statistically rather hand-waving. However, the paper by Jagannathan & Wang implies that its authors employ the same method. Another method, which is also not entirely correct because it omits the correlation between results of subsequent months, is calculating the standard deviation by summing the variances. These results are given in the table as 't-value via VAR'.

For the other variables, similar t-significances are achieved as by Jagannathan & Wang, although the c-values themselves differ substantially. This is a result of the different

indicators used for the risk premium as well as the labor income. The c_{vw} is lower by a factor of 100, which indicates that Jagannathan & Wang have done the calculations using percentages, while we employed rational numbers.
For the CAPM model, we achieve an R^2 of 23.29. This value is impressively significant, especially compared to the R^2 of 1.35 calculated in Jagannathan & Wang. However, when adjusting for size, Jagannathan & Wang obtain an R^2 of 57.56. Hence, we can explain the high value of our R^2 by the sensible selection of similarly-sized firms provided by the MDAX, which already yields a value half as high as when adjusting for size specifically. Another contributing factor is the use of the MDAX as index for the total market portfolio: of course the stocks listed in the MDAX itself will have a high correlation with the MDAX.
The results for the PLM improve to an R^2 of 55.21, a value similar to those acquired by Jagannathan & Wang. The t-values of both, c_{prem} and c_{labor} are significant compared to the values of the variables itself. The large increase in R^2 confirms the superiority of the PLM over the CAPM.
For a visual intuition, the two charts given on the next page present the realized yearly returns of each portfolio with respect to the expected yearly returns of each portfolio of each year from 2005 to 2015, with the CAPM and the PLM respectively. That is, the monthly returns calculated by the cross-sectional regressions are grouped together for each year. The red line is the 45° straight line from the origin.
As we can see, even the standard CAPM delivers a surprisingly accurate prediction of the yearly stock returns. We also see one single portfolio with predicted as well as realized returns of above 100%, which interestingly is from last year, 2015.
The PLM improves the predictions made by CAPM, with the portfolios visibly moving closer to the red diagonal. The graph demonstrates the accurateness of even the simple assumptions made by the PLM. Also visible is the different composition of the portfolios between the two models, as the β_{vw} slightly change.

6. Conclusion

The empirical results of both, the CAPM and the PLM show that even with simple assumptions, an impressively accurate prediction of the stock returns can be achieved. However, due to the limitations of the data for this project, we can also conclude that these predictions strongly rely on filtering out the size effect, as well as choosing a closely related market index.
Furthermore, the PLM improves the predictions of the CAPM, and elegantly extends the model by using only two additional economic indices. The limitation of the income data to only quarterly reports does not seem to inhibit the model in itself.

All in all, the comparison with the papers analyzing the US stock market show a large similarity between the two markets. However, further research is required in order to single out specific differences. Especially the size effect may play a smaller role in the German market, since we were able to easily omit it in our calculations.